Widmark's Equation

During the early part of this century, E.M.P. Widmark, a Swedish physician, did much of the foundational research regarding alcohol pharmacokinetics in the human body. In addition, he developed an algebraic equation allowing one to estimate any one of six variables given the other five. Typically, we are interested in determining either the amount of alcohol consumed by an individual or the associated blood alcohol concentration (BAC) given the values of the other variables. According to Widmark's equation, the amount of alcohol consumed (A) is a function of these several variables:

$$N = f(W, r, C_t, \beta, t, z) \qquad \text{Equation 1}$$

where:
- N = amount consumed
- W = body weight
- r = the volume of distribution (a constant)
- C_t = blood alcohol concentration (BAC)
- β = the alcohol elimination rate
- t = time since the first drink
- z = the fluid ounces of alcohol per drink

Widmark's equation relates these variables according to:

$$N = \frac{Wr(C_t + \beta t)}{0.8z} \qquad \text{Equation 2}$$

where:
- N = the number of drinks consumed
- W = body weight in ounces
- r = volume of distribution (a constant relating the distribution of water in the body in L/Kg)
- C_t = the blood alcohol concentration (BAC) in Kg/L
- β = the alcohol elimination rate in Kg/L/hr
- t = time since the first drink in hours
- z = the fluid ounces of alcohol per drink
- 0.8 = the density of ethanol (0.8 oz. per fluid ounce)

Example 1

Assume that we are interested in determining the amount of alcohol consumed (number of drinks) given certain information. The information we are given includes: a male weighing 185 lbs., r = 0.68 L/Kg, C_t = 0.15 g/100ml, β = 0.015 g/100ml/hr, t = 5 hours, and drinking 12 fl.oz. beers with 4% alcohol by volume. We introduce this information into Equation 2 according to:

$$N = \frac{(180\text{lb})(16\text{oz/lb})(0.68\text{L/Kg})(0.0015\text{Kg/L} + (0.00015\text{Kg/L/hr})(5\text{hr}))}{(0.8)(0.48\text{fl.oz./drink})}$$

Notice that we had to convert the 0.15 g/100ml and the 0.015 g/100ml/hr to Kg/L which simply amounts to moving the decimal two places to the left. Solving for A we find:

$$N = \frac{1958.4(0.00225)}{0.384} = 11.5\,\text{drinks}$$

Example 2

The next most common use of Widmark's equation is to determine the blood alcohol concentration (C_t) given the number of drinks consumed. We now assume the following: a female weighing 125 lbs., r = 0.55 L/Kg, β = 0.017 g/100ml/hr, t = 4 hours, and consumed 7 one fluid ounce glasses of 80 proof vodka. Employing Equation 1 above we introduce the information we are given and solve for C_t as follows:

$$7 = \frac{(125\text{lb})(16\text{oz/lb})(0.55\text{L/Kg})(C_t + (0.00017\text{Kg/L/hr})(4\text{hr}))}{(0.8)(0.40\text{fl.oz./drink})}$$

Notice that we are interested in finding C_t so we rearrange this equation as follows:

$$2.24 = 1100(C_t + 0.00068)$$

$$C_t = 0.00136 \text{ Kg/L} = 0.136 \text{ g/100ml}$$

Uncertainty In Widmark Estimates

Since each variable introduced into Widmark's equation is subject to measurement uncertainty, the computed values will also have uncertainty propagated through the computations. Widmark developed two other equations (one for men and one for women) where he was able to estimate the uncertainty (one standard deviation) in his computed value for N. His uncertainty equations are (1):

$$\text{men} \qquad S_N = \sqrt{0.015625\,N^2 + 0.050176(N \;\; (0.68\,C_t\,W/0.8z))^2}$$

$$\text{women} \qquad S_N = \sqrt{0.01\,N^2 + 0.021904(N \;\; (0.55\,C_t\,W/0.8z))^2}$$

Applying the equation for men to our Example 1 above we obtain:

$$S_N = \sqrt{0.015625(11.5)^2 + 0.050176((11.5) \;\; (0.68)(0.0015)(180)(16)/(0.8)(0.48))^2}$$

Solving for S_N we obtain:

$$S_N = 1.7 \text{ drinks}$$

This indicates that the one standard deviation uncertainty for N is 1.7 drinks and the more commonly employed two standard deviation estimate would be 3.4 drinks. Thus, our estimate for the number of drinks in Example 1 above should properly be stated as:

11.5 ± 3.4 drinks

This is a large and overly conservative estimate of the error in N. Other work (2,3) suggests that a better estimate of the two standard deviation estimate in N is closer to ± 20%. For Example 1 the better estimate would be:

11.5 ± 2.3 drinks

Using Breath Alcohol Results in Widmark's Equation

Most forensic cases utilizing Widmark's equation will employ breath alcohol rather than blood alcohol results. When doing so, the breath alcohol concentration (BrAC) must be converted to an estimated blood alcohol concentration (BAC) before introducing the value into the equation. This introduces another factor having uncertainty - the BAC/BrAC conversion factor. Typically, the breath and corresponding blood alcohol values are assumed to be the same (using a conversion factor of $K_{bac/brac} = 2100$). This is reasonable in forensic cases since it will typically benefit the defendant by providing an underestimate of their true BAC by substituting their BrAC. These principles should be kept in mind when working with Widmark's equation.

References

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